

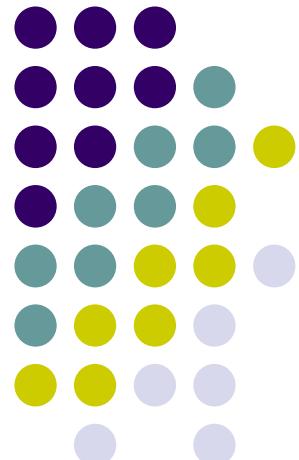
وزارت علوم، تحقیقات و فناوری



برنامه ریزی حمل و نقل

توزيع سفر

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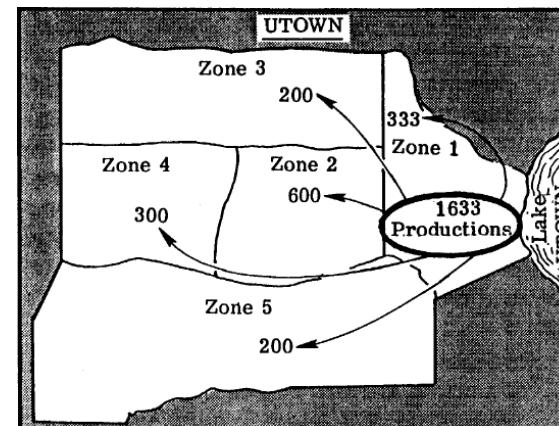
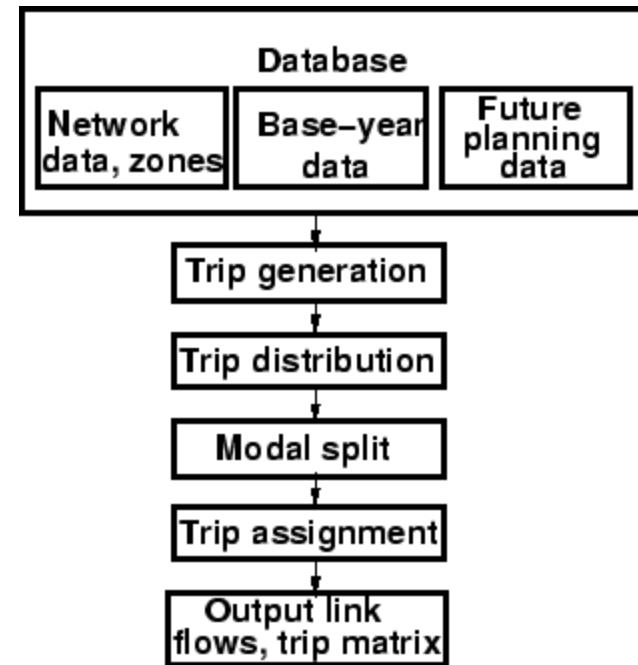


Overview

- The second stage of travel demand modeling
- Trips from each zone to all other zones (choice of destination)

methods of trip distribution:

- Growth factor model
- Fratar model
- Gravity model
- Other models (Intervening opportunities model , Destination choice models, Entropy model)



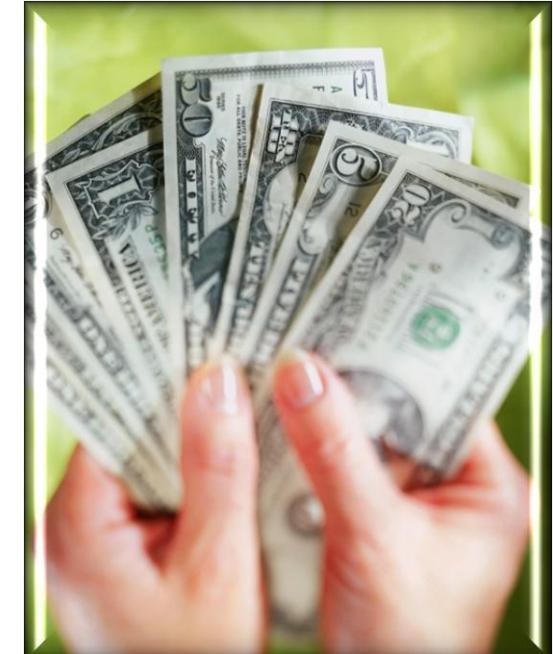
► Definitions and notations

- Trip matrix

Zones	1	2	...	j	...	n	O_i
1	T_{11}	T_{12}	...	T_{1j}	...	T_{1n}	O_1
2	T_{21}	T_{22}	...	T_{2j}	...	T_{2n}	O_2
\vdots	\vdots
	T_{i1}	T_{i2}	...	T_{ij}	...	T_{in}	O_i
\vdots	\vdots
n	T_{ni}	T_{n2}	...	T_{nj}	...	T_{nn}	O_n
D_j	D_1	D_2	...	D_j	...	D_n	T

- Generalized cost

$$C_{ij} = \alpha \cdot t_{ij}^\lambda + \beta \cdot F_{ij}^\gamma + \delta$$



Growth factor methods

- Uniform growth factor

☒ Example:

$$T_{ij} = f \times t_{ij}$$



	1	2	3	o_i
1	20	30	28	78
2	36	32	24	92
3	22	34	26	82
d_j	88	96	78	252

$$f=1.3$$



	1	2	3	O_i
1	26	39	36.4	101.4
2	46.8	41.6	31.2	119.6
3	28.6	44.2	33.8	106.2
D_j	101.4	124.8	101.4	327.6

Growth factor methods

- Doubly constrained growth factor model



$$T_{ij} = t_{ij} \times a_i \times b_j$$

$$E = \sum |O_i - O_i^1| + \sum |D_j - D_j^1|$$

Example:

	1	2	3	o_i
1	20	30	28	78 98
2	36	32	24	92 106
3	22	34	26	82 122
d_j	88	96	78	252 326

102 118 106



	1	2	3	o_i
1	25.2	37.8	35.28	98
2	41.4	36.8	27.6	106
3	32.78	50.66	38.74	122
d_j^1	99.38	125.26	101.62	
D_j	102	118	106	



	1	2	3	o_i	O_i
1	25.96	35.53	36.69	98.18	98
2	42.64	34.59	28.70	105.93	106
3	33.76	47.62	40.29	121.67	122
b_j	1.03	0.94	1.04		
D_j	102	118	106		

● Fratar Model

The total trips emanating from a zone are distributed to the interzonal movements and according to the relative attraction of each movement, locational factors for each zone are calculated. Then:

$$T_{ij}^1 = T_{ij}^0 F_i^0 F_j^0 \frac{L_i^0 + L_j^0}{2}$$

$$L_i^0 = \frac{\sum_{m=1}^n T_{im}^0}{\sum_{m=1}^n F_m^0 T_{im}}$$

$$L_j^0 = \frac{\sum_{m=1}^n T_{jm}^0}{\sum_{m=1}^n F_m^0 T_{jm}}$$



● Fratar Model

Example:

	A	B	C	D	$\sum_i T_{ij}$	F_i	t_{ij}
A	-	12	10	18	40	2	80
B	12	-	14	6	32	1.5	48
C	10	14	-	14	38	3	114
D	18	6	14	-	38	1	38
$\sum_j T_{ij}$	40	32	38	38	148	-	280
F_j	2	1.5	3	1	-	-	

$$L_i^0 = \frac{\sum_{m=1}^n T_{im}^0}{\sum_{m=1}^n F_m^0 T_{im}} \Rightarrow L_A^0 = \frac{40}{1.5 * 12 + 3 * 10 + 1 * 18} = \frac{40}{66}$$

$$L_j^0 = \frac{\sum_{m=1}^n T_{jm}^0}{\sum_{m=1}^n F_m^0 T_{jm}} \Rightarrow L_B^0 = \frac{32}{2 * 12 + 3 * 14 + 1 * 6} = \frac{32}{72}$$

$$T_{ij}^1 = T_{ij}^0 F_i^0 F_j^0 \frac{L_i^0 + L_j^0}{2} \Rightarrow T_{AB}^1 = 12 * 2 * 1.5 \frac{\frac{40}{66} + \frac{32}{72}}{2}$$



	A	B	C	D	$\sum_i T_{ij}$
A	-	18.9	38.9	18.8	76.6
B	18.9	-	35.8	4	58.6
C	38.9	35.8	-	23.7	98.4
D	18.8	4	23.7	-	46.4
$\sum_j T_{ij}$	76.6	58.6	98.4	46.4	280
F_i^1	1.04	0.82	1.16	0.82	

Gravity model

$$T_{ij} = CO_i D_j / c_{ij^n}$$

$$T_{ij} = A_i O_i B_j D_j f(c_{ij})$$

deterrence function $f(c_{ij}) = e^{-\beta c_{ij}}$

$$f(c_{OJ}) = c_{ij}^{-n}$$

$$f(c_{ij}) = c_{ij}^{-n} \times e^{-\beta c_{ij}}$$

$$\Sigma_i T_{ij} = \Sigma_i A_i O_i B_j D_j f(c_{ij})$$

$$\Sigma_i T_{ij} = D_j$$

$$D_j = B_j D_j \Sigma_i A_i O_i f(c_{ij})$$

$$B_j = 1 / \Sigma_i A_i O_i f(c_{ij})$$

$$A_i = 1 / \Sigma_j B_j D_j f(c_{ij})$$

Gravity model

Example:

$$\begin{array}{c}
 98 \\
 106 \\
 122 \\
 326 \\
 \\
 102 \quad 118 \quad 106
 \end{array}
 \qquad
 f(c_{ij}) = 1/c_{ij}^2
 \qquad
 \begin{bmatrix}
 1.0 & 1.2 & 1.8 \\
 1.2 & 1.0 & 1.5 \\
 1.8 & 1.5 & 1.0
 \end{bmatrix}$$

Table 8:1: Step1: Computation of parameter A_i

i	j	B_j	D_J	$f(c_{ij})$	$B_j D_j f(c_{ij})$	$\sum B_j D_j f(c_{ij})$	$A_i = \frac{1}{\sum B_j D_j f(c_{ij})}$
1	1	1.0	102	1.0	102.00	216.28	0.00462
	2	1.0	118	0.69	81.42		
	3	1.0	106	0.31	32.86		
2	1	1.0	102	0.69	70.38	235.02	0.00425
	2	1.0	118	1.0	118		
	3	1.0	106	0.44	46.64		
3	1	1.0	102	0.31	31.62	189.54	0.00527
	2	1.0	118	0.44	51.92		
	3	1.0	106	1.00	106		



Gravity model

Example:

$$\begin{array}{c}
 98 \\
 106 \\
 122 \\
 326 \\
 \\
 102 \quad 118 \quad 106
 \end{array}
 \quad f(c_{ij}) = 1/c_{ij}^2 \quad \left[\begin{array}{ccc} 1.0 & 1.2 & 1.8 \\ 1.2 & 1.0 & 1.5 \\ 1.8 & 1.5 & 1.0 \end{array} \right]$$

Table 8:2: Step2: Computation of parameter B_j

j	i	A_i	O_i	$f(c_{ij})$	$A_i O_i f(c_{ij})$	$\sum A_i O_i f(c_{ij})$	$B_j = 1 / \sum A_i O_i f(c_{ij})$
1	1	0.00462	98	1.0	0.4523	0.9618	1.0397
	2	0.00425	106	0.694	0.3117		
	3	0.00527	122	0.308	0.1978		
2	1	0.00462	98	0.69	0.3124	1.0458	0.9562
	2	0.00425	106	1.0	0.4505		
	3	0.00527	122	0.44	0.2829		
3	1	0.00462	98	0.31	0.1404	0.9815	1.0188
	2	0.00425	106	0.44	0.1982		
	3	0.00527	122	1.00	0.6429		



Gravity model

Example:

$$\begin{array}{c}
 \begin{matrix} 98 \\ 106 \\ 122 \\ 326 \end{matrix} \\
 f(c_{ij}) = 1/c_{ij}^2 \quad \begin{bmatrix} 1.0 & 1.2 & 1.8 \\ 1.2 & 1.0 & 1.5 \\ 1.8 & 1.5 & 1.0 \end{bmatrix} \\
 \begin{matrix} 102 & 118 & 106 \end{matrix} \\
 T_{ij} = A_i O_i B_j D_j f(c_{ij}) \\
 T_{11} = 102 \times 1.0397 \times 0.00462 \times 98 \times 1 = 48.01
 \end{array}$$

Table 8:3: Step3: Final Table

	1	2	3	A_i	O_i	O_i^1
1	48.01	35.24	15.157	0.00462	98	98.407
2	32.96	50.83	21.40	0.00425	106	105.19
3	21.14	31.919	69.43	0.00527	122	122.489
B_j	1.0397	0.9562	1.0188			
D_j	102	118	106			
D_j^1	102.11	117.989	105.987			

$$\begin{aligned}
 Error = \sum |O_i - O_i^1| + \sum |D_j - D_j^1| \quad Error = |98 - 98.407| + |106 - \\
 105.19| + |122 - 122.489| + |102 - 102.11| + |118 - 117.989| + |106 - 105.987| = 2.03
 \end{aligned}$$