

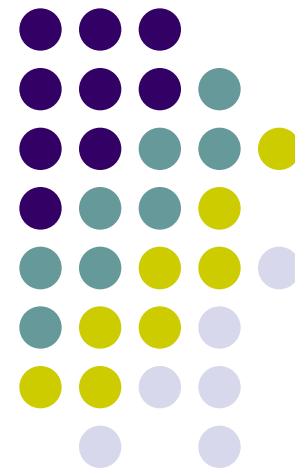
سازره های نو

وزارت علوم، تحقیقات و فناوری



دانشگاه سوره

تئوری سازه های کابلی

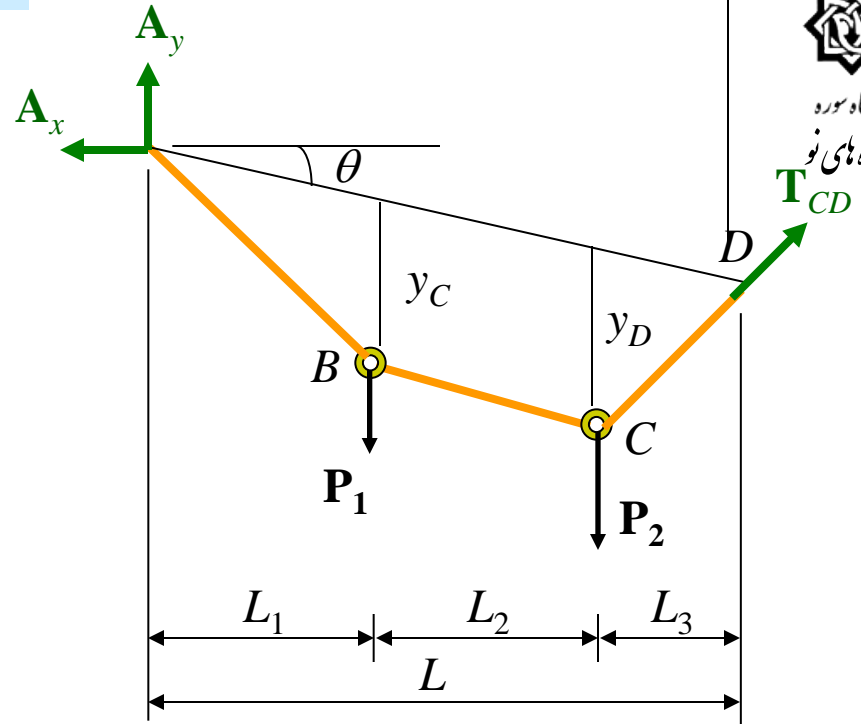
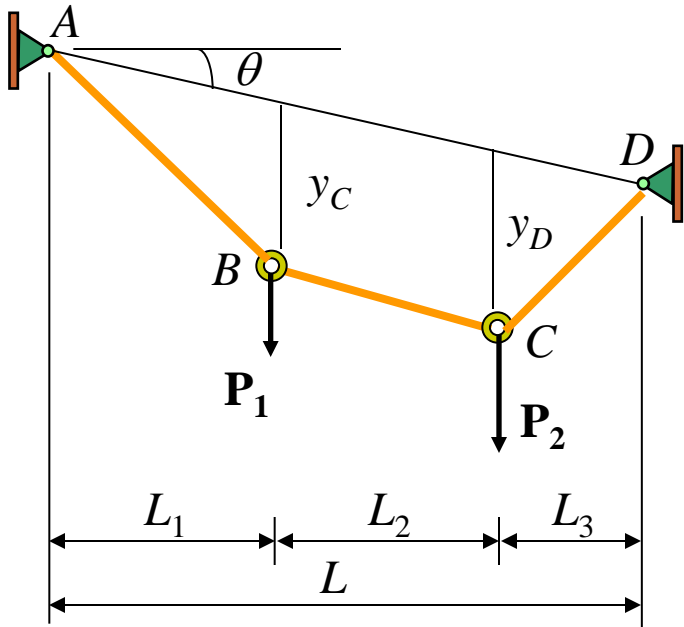




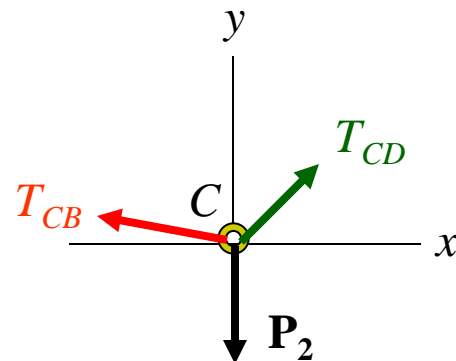
Introduction

- Cables are pure tension members.
- Used as
 - Supports to suspension roofs
 - Suspension bridges
- Self weight of cable is neglected in analysis of above structures
- When used as guys for antennas or transmission lines, weight is considered.

Cable Subjected to Concentrated Loads

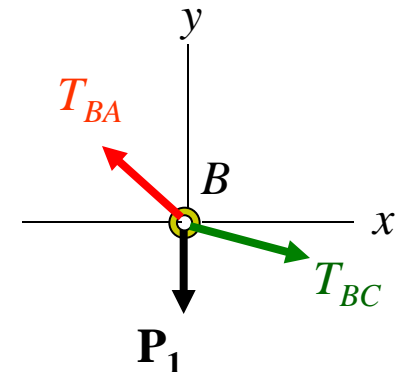


$\curvearrowright \Sigma M_A = 0:$
Obtain T_{CD}



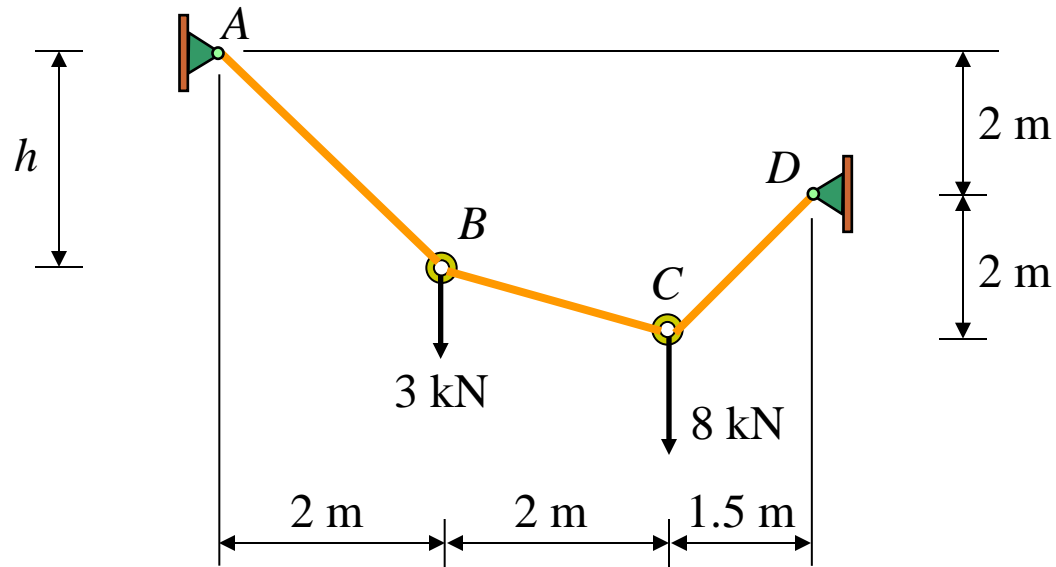
$\rightarrow \Sigma F_x = 0:$

$\uparrow \Sigma F_y = 0:$

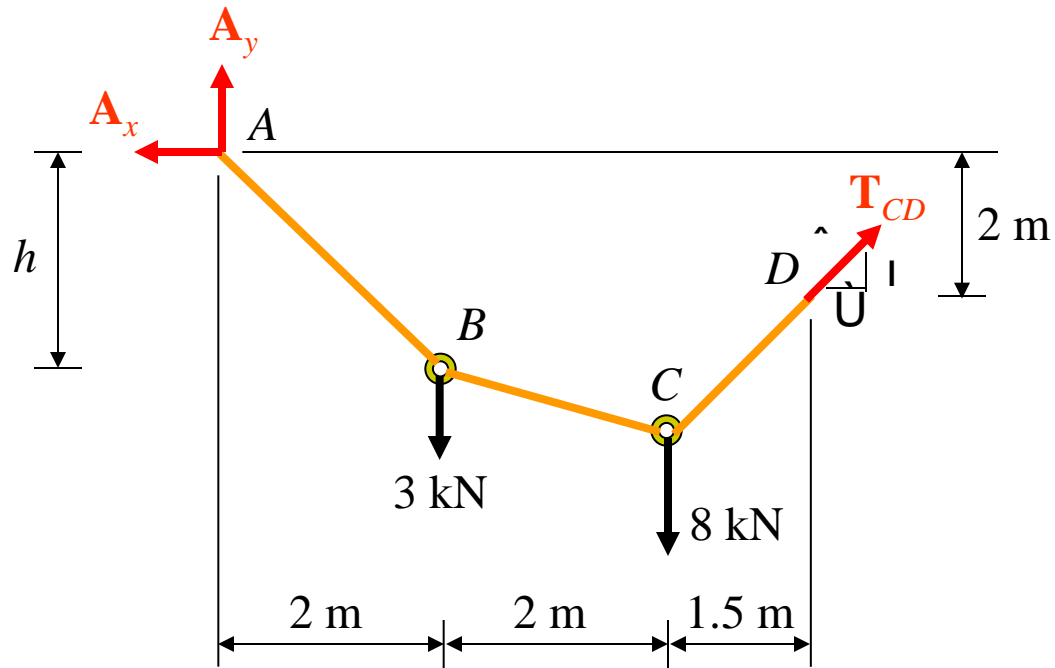


Example 1

Determine the tension in each segment of the cable shown in the figure below.
Also, what is the dimension h ?



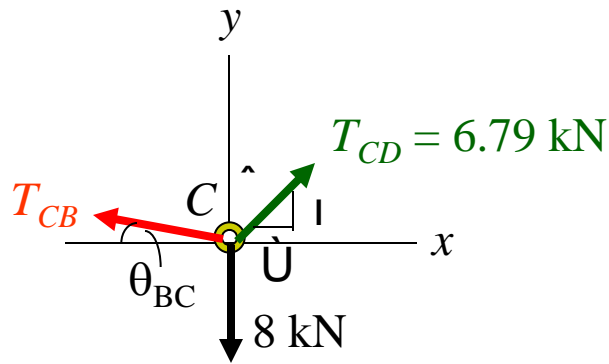
SOLUTION



$$+\curvearrowright \Sigma M_A = 0:$$

$$T_{CD}(3/5)(2 \text{ m}) + T_{CD}(4/5)(5.5 \text{ m}) - 3 \text{ kN}(2 \text{ m}) - 8 \text{ kN}(4 \text{ m}) = 0$$

$$T_{CD} = 6.79 \text{ kN}$$

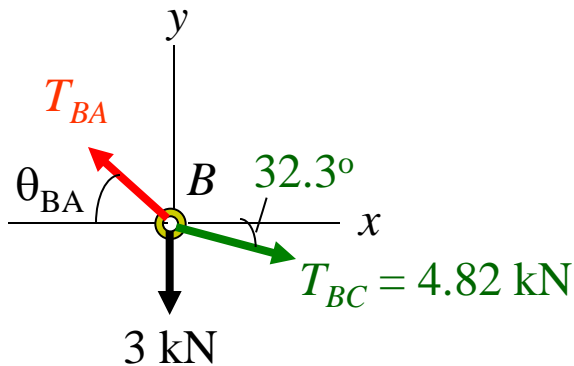


Joint C

$$\rightarrow \Sigma F_x = 0: 6.79(3/5) - T_{CB} \cos \theta_{BC} = 0$$

$$\uparrow \Sigma F_y = 0: 6.79(4/5) - 8 + T_{CB} \sin \theta_{CB} = 0$$

$$\theta_{BC} = 32.3^\circ \quad T_{CB} = 4.82 \text{ kN}$$



Joint B

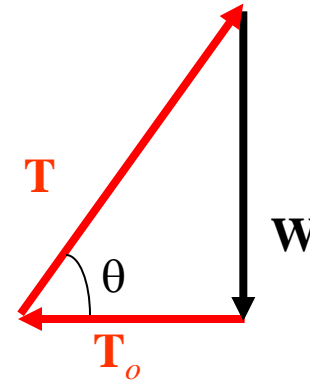
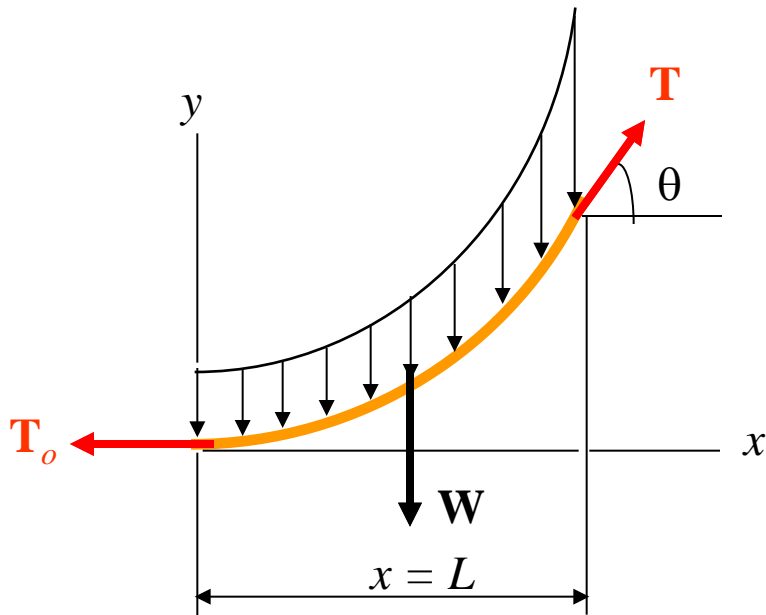
$$\rightarrow \Sigma F_x = 0: -T_{BA} \cos \theta_{BA} + 4.82 \cos 32.3^\circ = 0$$

$$\uparrow \Sigma F_y = 0: T_{BA} \sin \theta_{BA} - 4.82 \sin 32.3^\circ - 3 = 0$$

$$\theta_{BA} = 53.8^\circ \quad T_{BA} = 6.90 \text{ kN}$$

$$h = 2 \tan \theta_{BA} = 2 \tan 53.8^\circ = 2.74 \text{ m}$$

Cable Subjected to Distributed Load



$$T \cos \theta = T_o = F_H = \text{Constant}$$

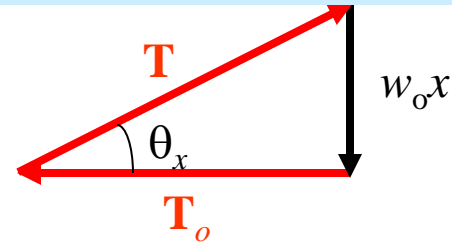
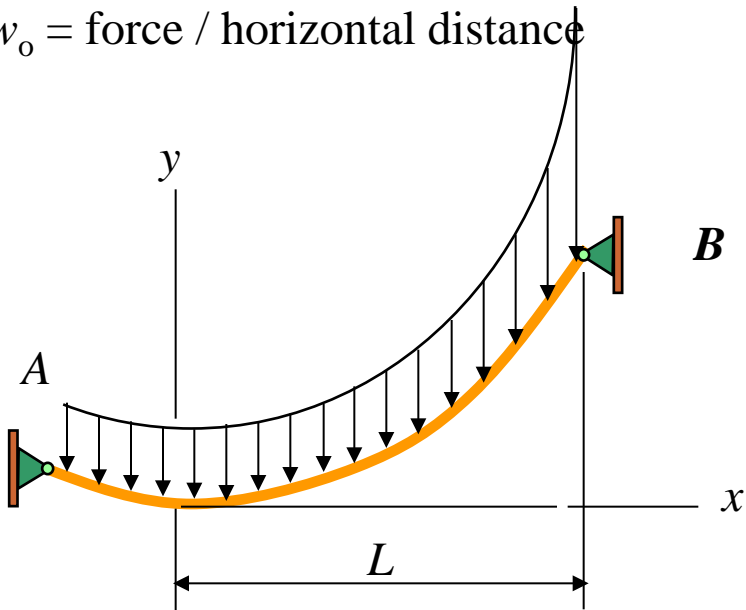
$$T \sin \theta = W$$

$$\frac{dy}{dx} = \tan \theta = \frac{W}{T_o}$$

Parabolic Cable: Subjected to Linear Uniform distributed Load



$w_o = \text{force} / \text{horizontal distance}$



$$\frac{dy}{dx} = \tan \theta = \frac{w_o x}{T_o}$$

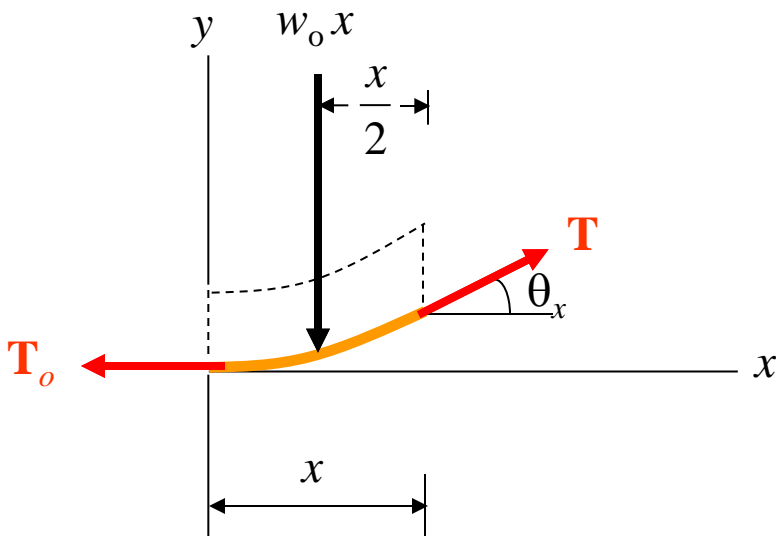
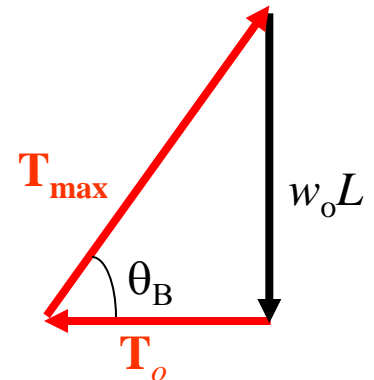
$$y = \int \frac{w_o x}{T_o} dx$$

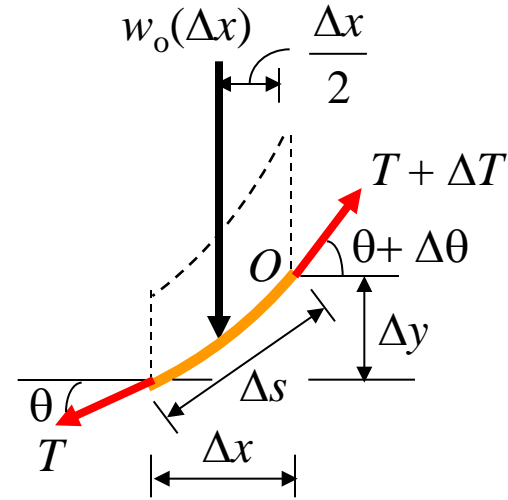
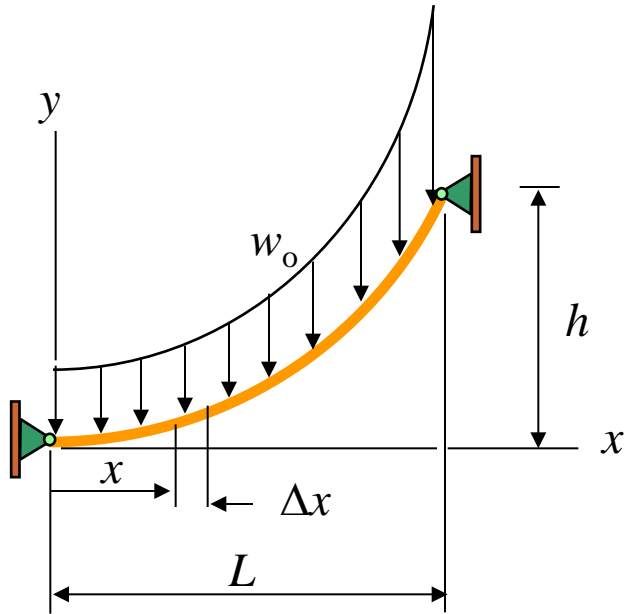
$$y = \frac{w_o x^2}{2T_o} + C_1$$

$$T_o = \frac{w_o x^2}{2y}$$

at $x = L$, $T = T_B = T_{\max}$

$$T_{\max} = \sqrt{T_o^2 + (w_o L)^2}$$





$$\pm \rightarrow \Sigma F_x = 0:$$

$$-T \cos \theta + (T + \Delta T) \cos (\theta + \Delta \theta) = 0$$

$$+\uparrow \Sigma F_y = 0:$$

$$-T \sin \theta - w_o(\Delta x) + (T + \Delta T) \sin (\theta + \Delta \theta) = 0$$

$$+\curvearrowright \Sigma M_O = 0:$$

$$w_o(\Delta x)(\Delta x/2) - T \cos \theta \Delta y + T \sin \theta(\Delta x) = 0$$



Dividing each of these equations by Δx and taking the limit as $\Delta x \rightarrow 0$, and hence $\Delta y \rightarrow 0$, $\Delta\theta \rightarrow 0$, and $\Delta T \rightarrow 0$, we obtain

$$\frac{d(T \cos \theta)}{dx} = 0 \quad \text{-----(1)}$$

$$\frac{d(T \sin \theta)}{dx} = w_o \quad \text{-----(2)}$$

$$\frac{dy}{dx} = \tan \theta \quad \text{-----(3)}$$

Integrating Eq. 1, where $T = F_H$ at $x = 0$, we have:

$$T \cos \theta = F_H \quad \text{-----(4)}$$

Integrating Eq. 2, where $T \sin \theta = 0$ at $x = 0$, gives

$$T \sin \theta = w_o x \quad \text{-----(5)}$$

Dividing Eq. 5 Eq. 4 eliminates T . Then using Eq. 3, we can obtain the slope at any point,

$$\tan \theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \quad \text{-----(6)}$$



Performing a second integration with $y = 0$ at $x = 0$ yields

$$y = \frac{w_o}{2F_H} x^2 \quad \text{-----}(7)$$

This is the equation of a *parabola*. The constant F_H may be obtained by using the boundary condition $y = h$ at $x = L$. Thus,

$$F_H = \frac{w_o L^2}{2h} \quad \text{-----}(8)$$

Finally, substituting into Eq. 7 yields

$$y = \frac{h}{L^2} x^2 \quad \text{-----}(9)$$

From Eq. 4 , the maximum tension in the cable occurs when θ is maximum; i.e., at $x = L$. Hence, from Eqs. 4 and 5,

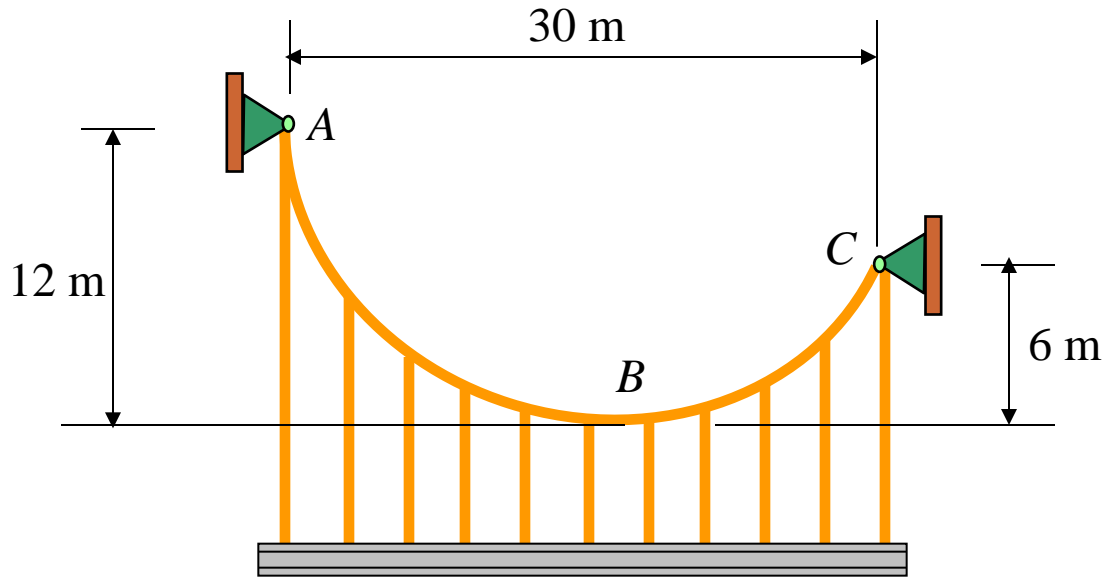
$$T_{\max} = \sqrt{F_H^2 + (w_o L)^2} \quad \text{-----}(10)$$

Or, using Eq. 8 , we can express T_{\max} in terms of w_o , i.e.,

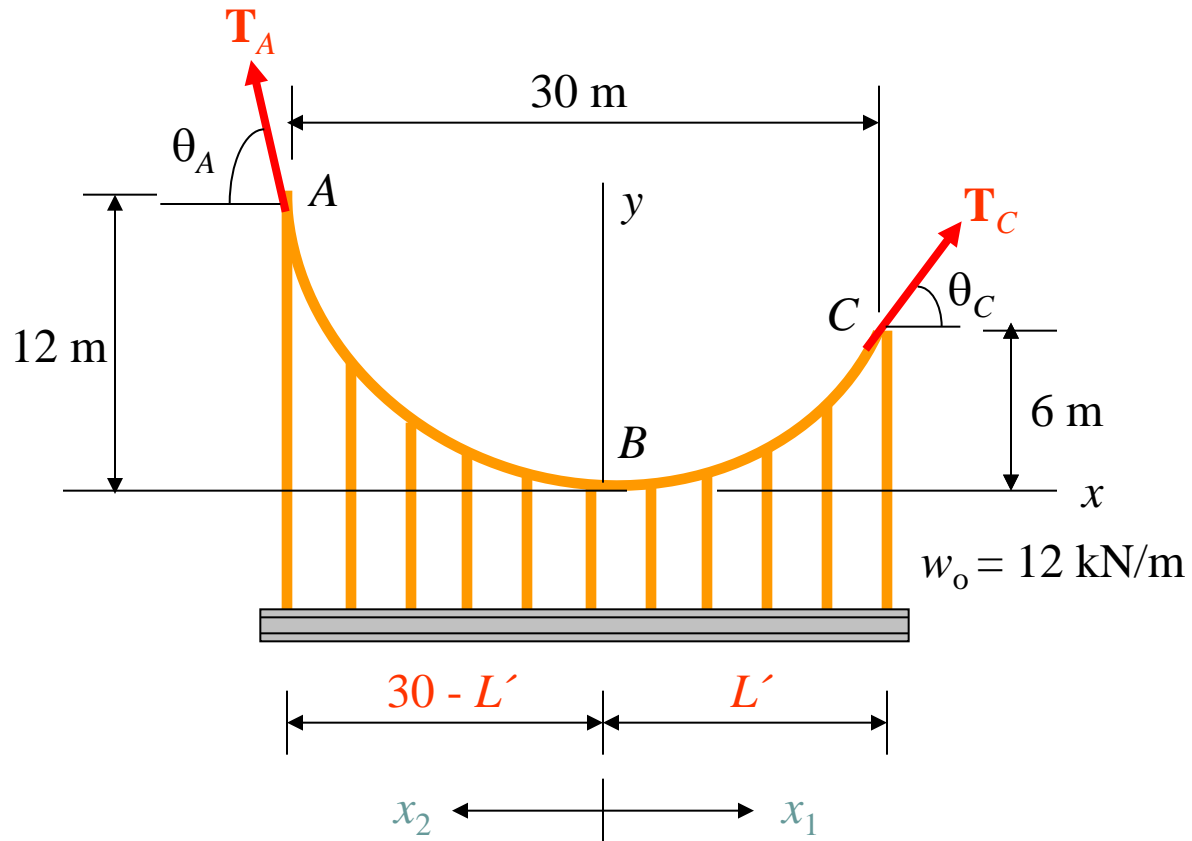
$$T_{\max} = w_o L \sqrt{1 + (L/2h)^2} \quad \text{-----}(11)$$

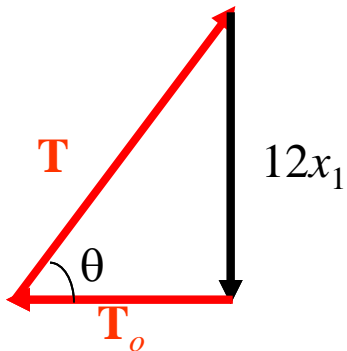
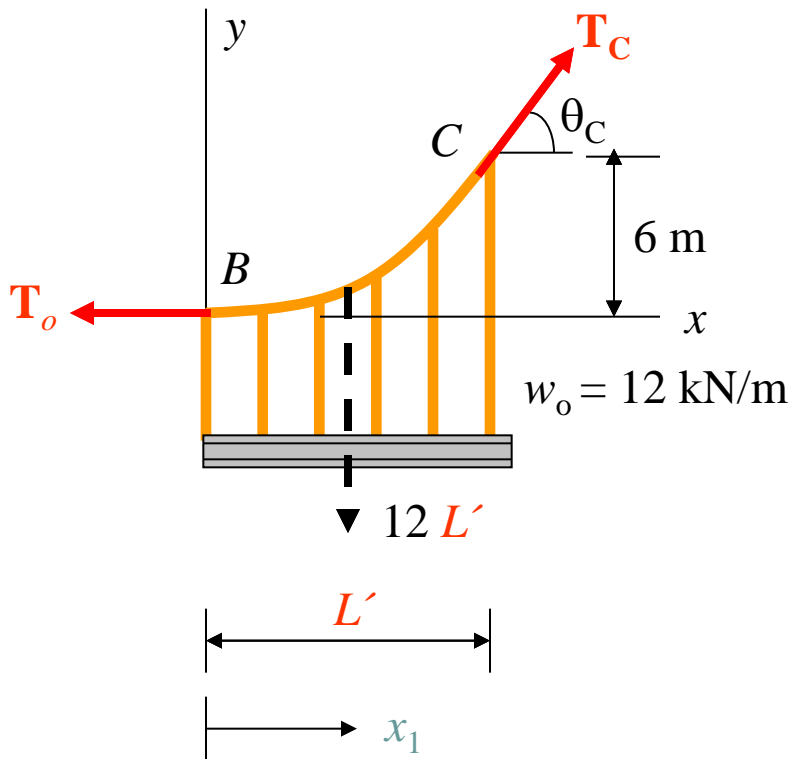
Example 2

The cable shown supports a girder which weighs 12kN/m . Determine the tension in the cable at points A , B , and C .



SOLUTION





$$\frac{dy_1}{dx_1} = \tan \theta = \frac{12x_1}{T_o}$$

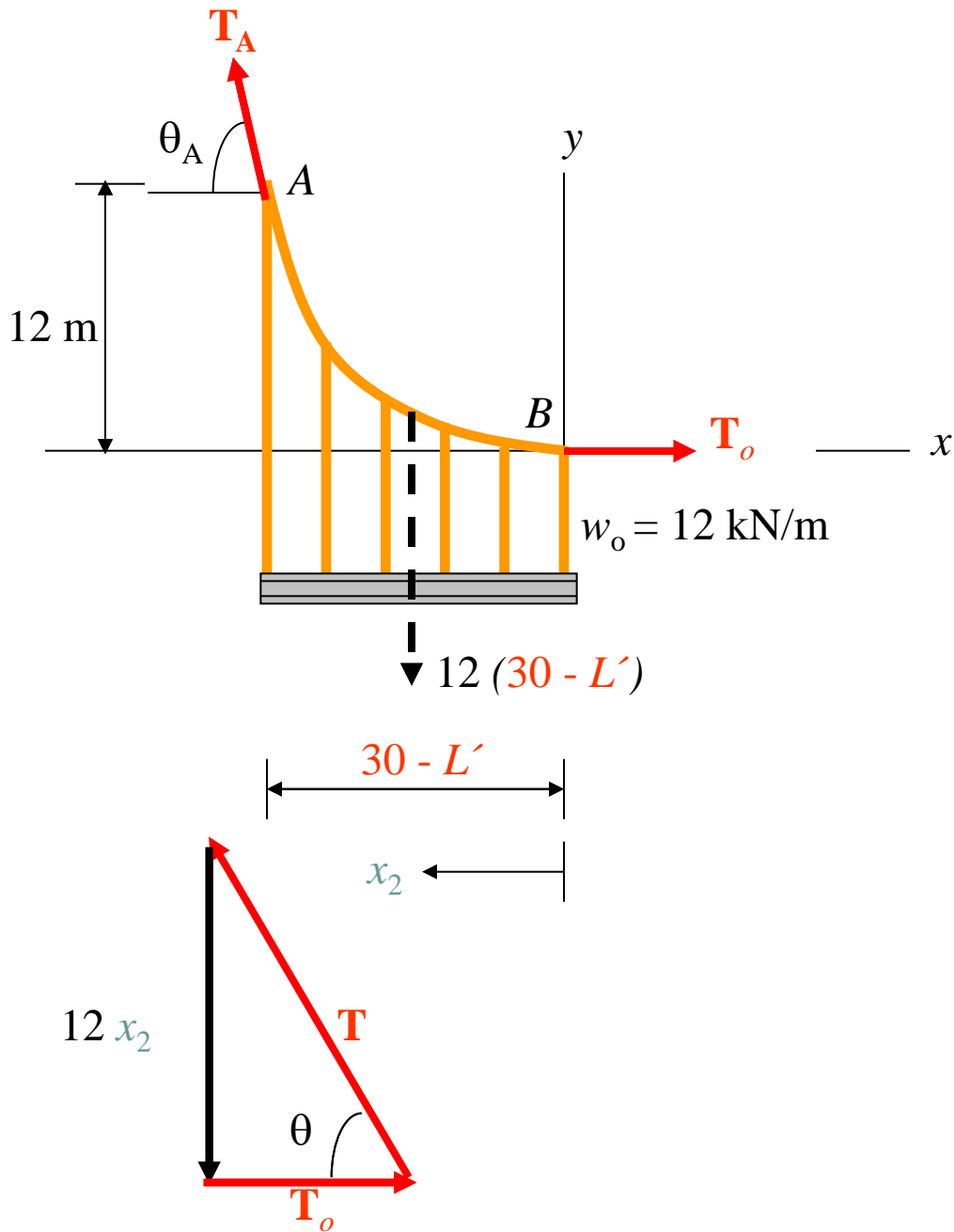
$$y_1 = \int \frac{12x_1}{T_o} dx_1$$

$$6 = \int_0^{L'} \frac{12x_1}{T_o} dx_1$$

$$6 = \frac{12x_1^2}{2T_o} \Big|_0^{L'} + C_1$$

$$6 = \frac{12L'^2}{2T_o}$$

$$T_o = L'^2 \quad \text{-----(1)}$$



$$\frac{dy_2}{dx_2} = \tan \theta = \frac{12x_2}{T_o}$$

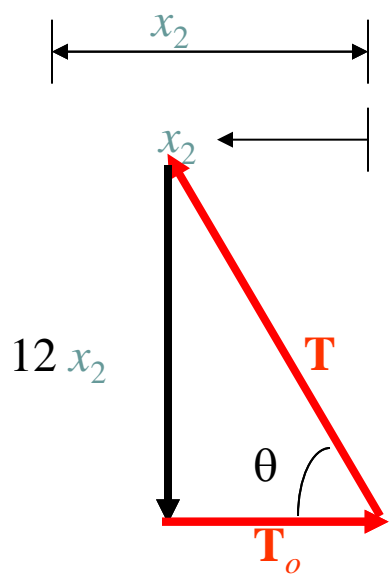
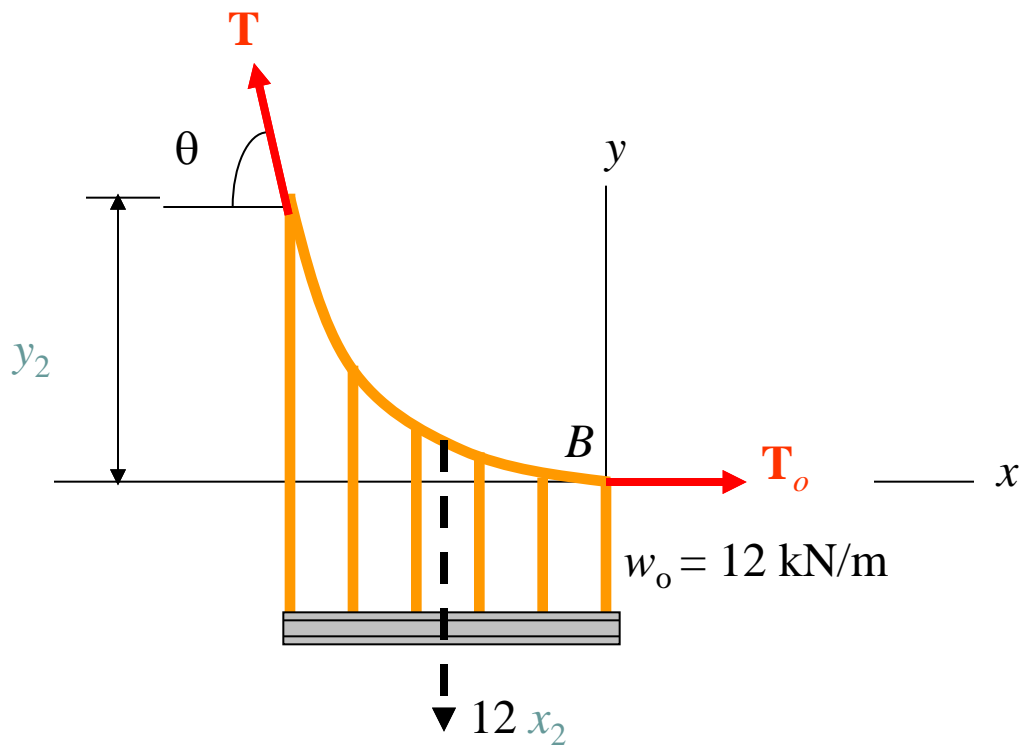
$$y_2 = \int \frac{12x_2}{T_o} dx_2$$

$$12 = \int_0^{(30-L')} \frac{12x_2}{T_o} dx_2$$

$$12 = \frac{12x_2^2}{2T_o} \Big|_0^{(30-L')} + C_2$$

~~$$12 = \frac{12(30-L')^2}{2T_o}$$~~

$$1 = \frac{(30-L')^2}{2T_o} \text{ -----(2)}$$



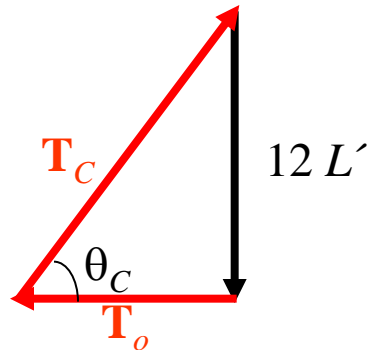


$$T_o = L'^2 \quad \text{-----(1)}$$

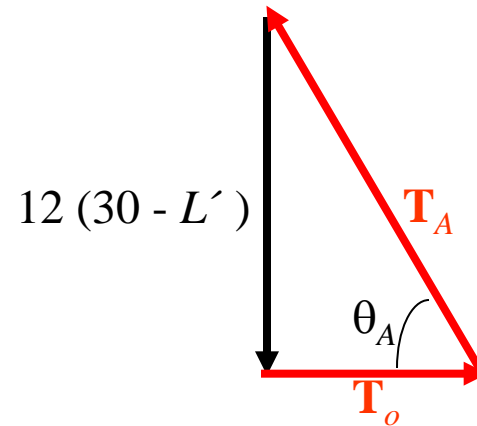
$$1 = \frac{(30 - L')^2}{2T_o} \quad \text{-----(2)}$$

From (1) and (2), $L' = 12.43 \text{ m}$, $T_o = 154.50 \text{ kN}$

$$T_B = T_o = 154.50 \text{ kN}$$



$$\begin{aligned} T_C &= \sqrt{T_o^2 + (12L')^2} \\ &= \sqrt{(154.50)^2 + (12 \times 12.43)^2} \\ &= 214.75 \text{ kN} \end{aligned}$$



$$\begin{aligned} T_A &= \sqrt{T_o^2 + [12(30 - L')]^2} \\ &= \sqrt{(154.50)^2 + [12(30 - 12.43)]^2} \\ &= 261.39 \text{ kN} \end{aligned}$$