وزارت علوم، تحقیقات و فناوری

دانشگاه سوره

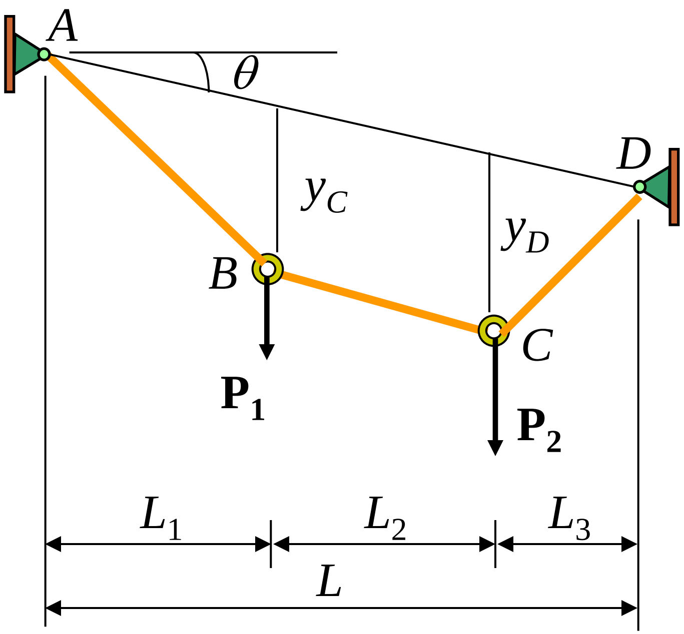
سازه های نو

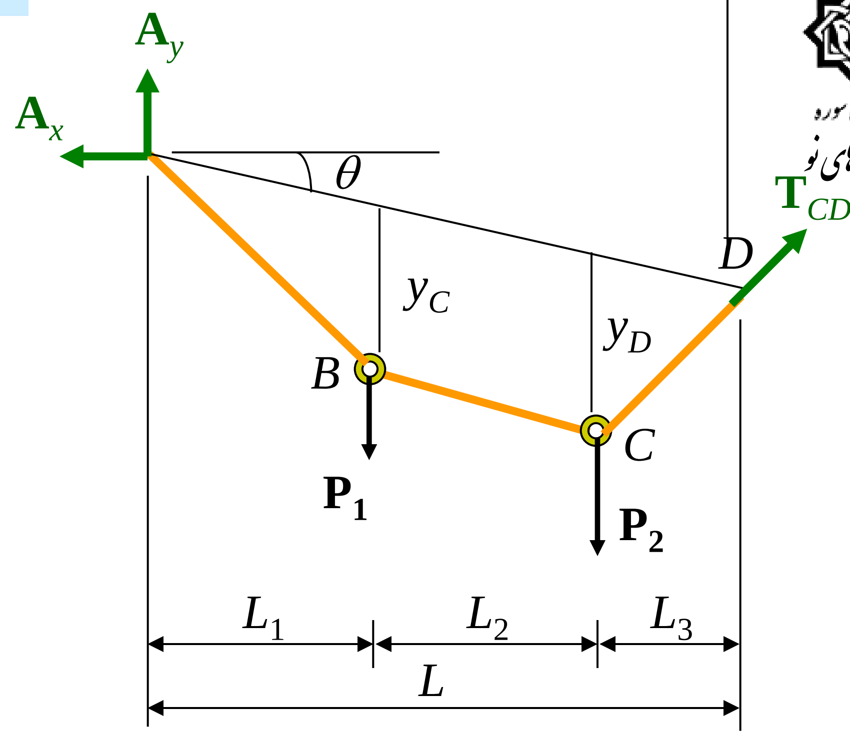
تئوری سازه های کابلی

Introduction

- Cables are pure tension members.
- Used as
  - Supports to suspension roofs
  - Suspension bridges
- Self weight of cable is neglected in analysis of above structures
- When used as guys for antennas or transmission lines, weight is considered.

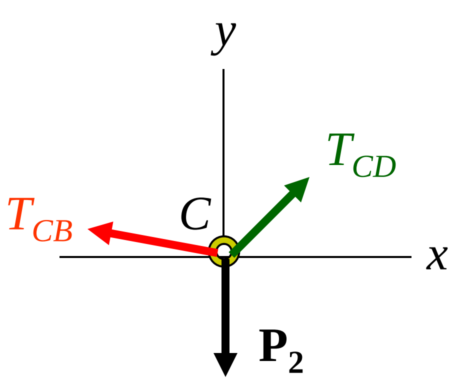
Cable Subjected to Concentrated Loads

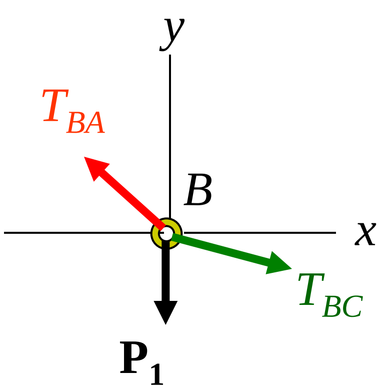




$+\circlearrowleft \; \Sigma M_A = 0$:

Obtain T_{CD}



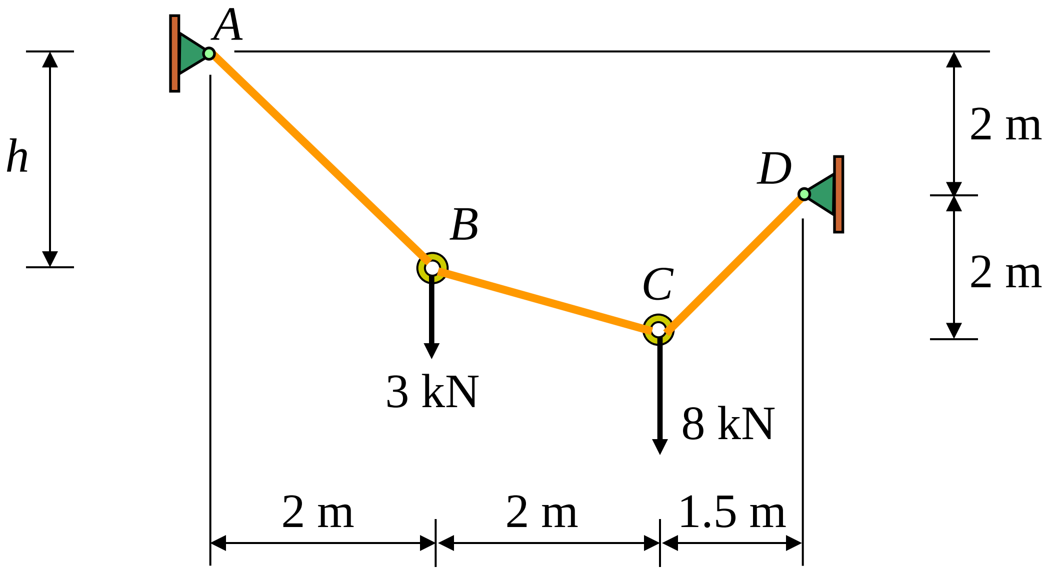


$\xrightarrow{+} \; \Sigma F_x = 0$:

$+\uparrow \; \Sigma F_y = 0$:

Example 1

Determine the tension in each segment of the cable shown in the figure below. Also, what is the dimension *h* ?



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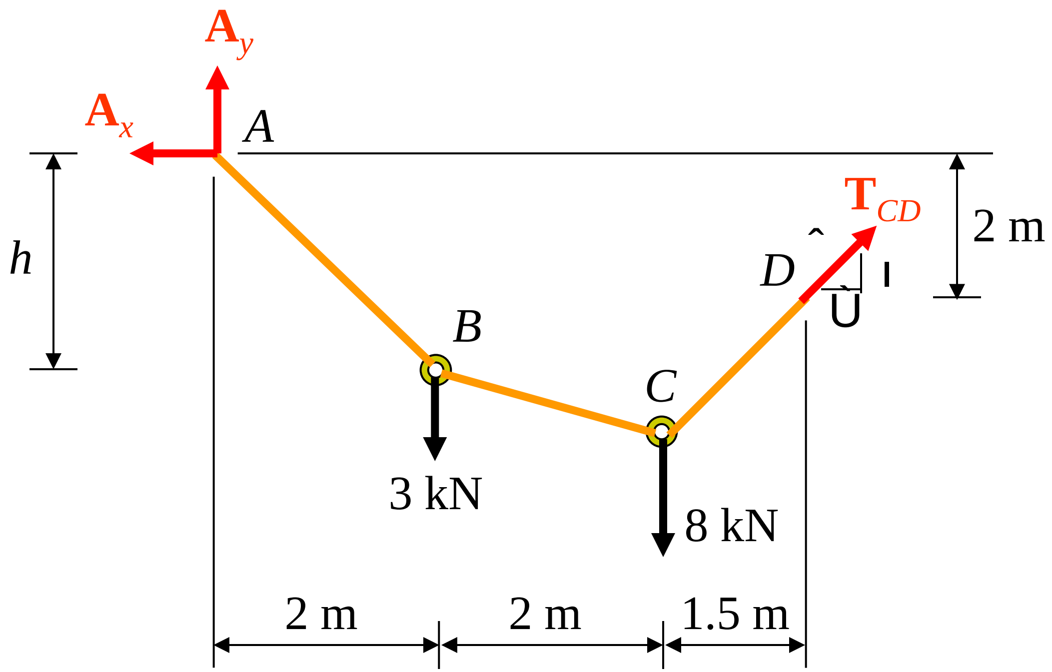
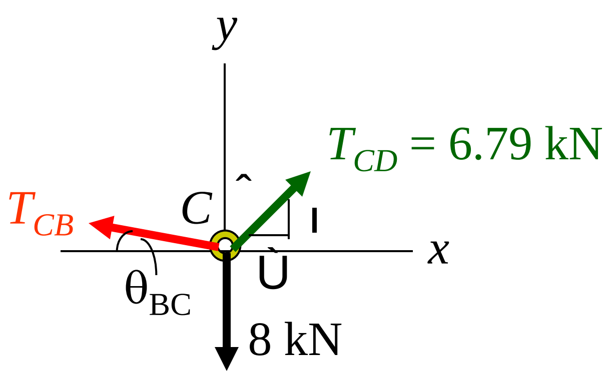
SOLUTION



$\circlearrowleft + \; \Sigma M_A = 0:$

$$T_{CD}(3/5)(2\text{ m}) + T_{CD}(4/5)(5.5\text{ m}) - 3\text{kN}(2\text{ m}) - 8\text{ kN}(4\text{ m}) = 0$$

$$T_{CD} = 6.79\text{ kN}$$

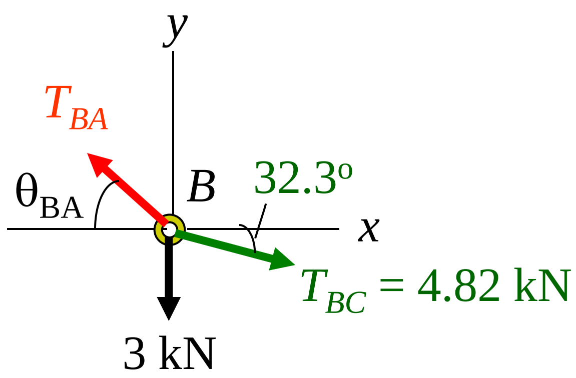


Joint C

$\xrightarrow{+}$ $\Sigma F_x = 0$: $6.79(3/5) - T_{CB}\cos\theta_{BC} = 0$

$+\uparrow$ $\Sigma F_y = 0$: $6.79(4/5) - 8 + T_{CB}\sin\theta_{CB} = 0$

$\theta_{BC} = 32.3^o$ $T_{CB} = 4.82$ kN



Joint B

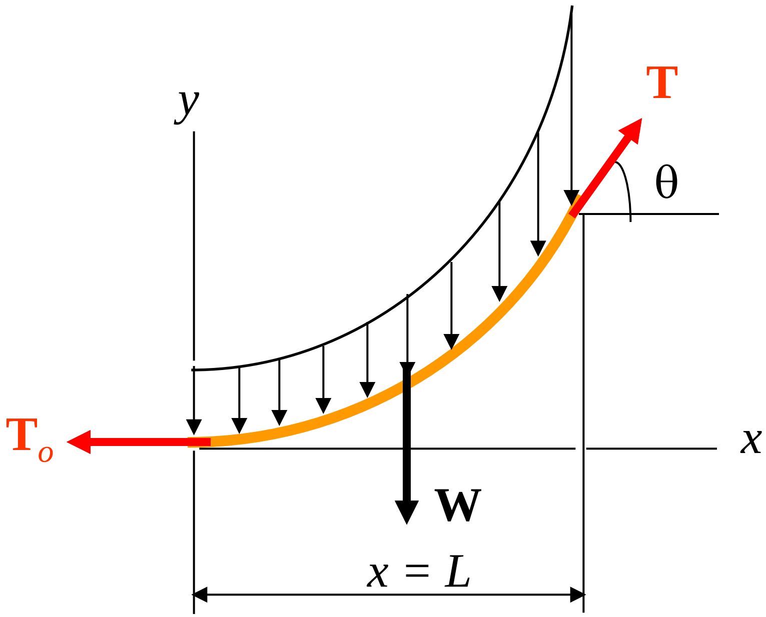
$\xrightarrow{+}$ $\Sigma F_x = 0$: $-T_{BA}\cos\theta_{BA} + 4.82\cos 32.3^o = 0$

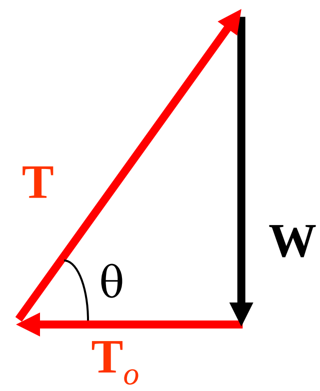
$+\uparrow$ $\Sigma F_y = 0$: $T_{BA}\sin\theta_{BA} - 4.82\sin 32.3^o - 3 = 0$

$\theta_{BA} = 53.8^o$ $T_{BA} = 6.90$ kN

$h = 2\tan\theta_{BA} = 2\tan 53.8^o = 2.74$ m

Cable Subjected to Distributed Load





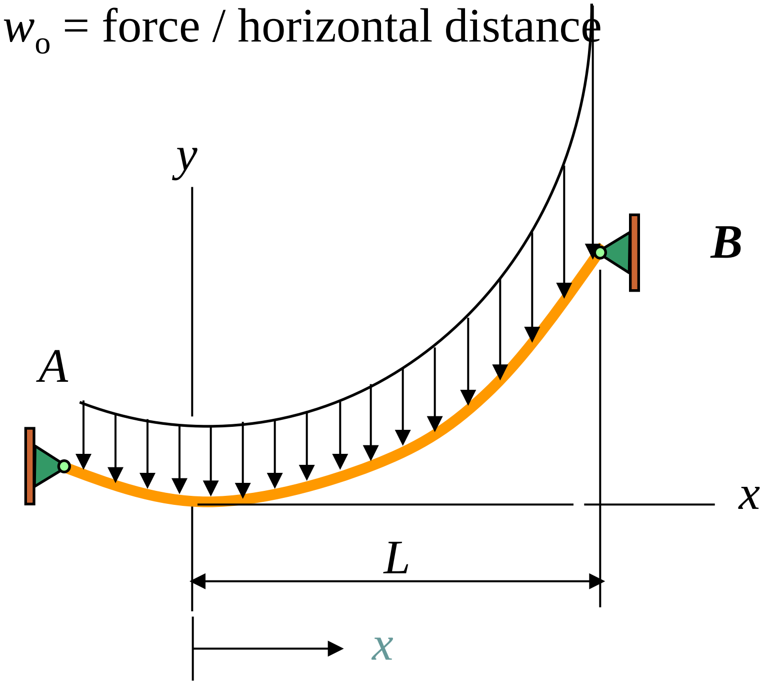
$$T \cos\theta = T_o = F_H = \text{Constant}$$

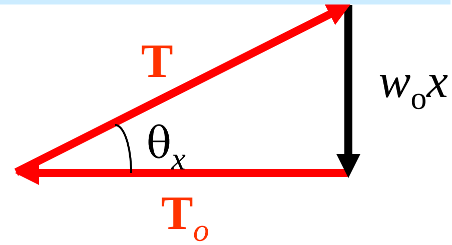
$$T \sin\theta = W$$

$$\frac{dy}{dx} = \tan\theta = \frac{W}{T_o}$$

Parabolic Cable: Subjected to Linear Uniform distributed Load

w_o = force / horizontal distance

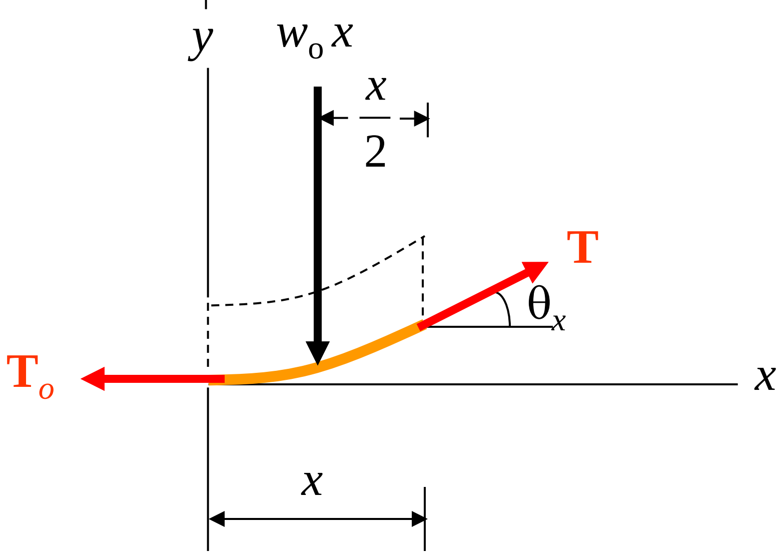




$$\frac{dy}{dx} = \tan\theta = \frac{w_o x}{T_o}$$

$$y = \int \frac{w_o x}{T_o} dx$$

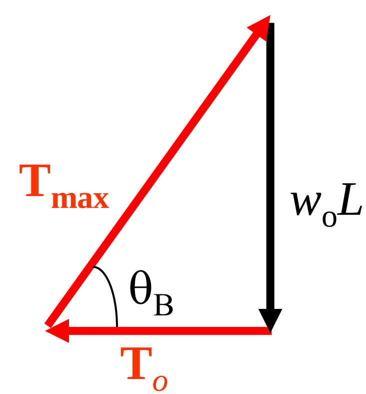
$$y = \frac{w_o x^2}{2T_o} + \cancel{C_1}$$

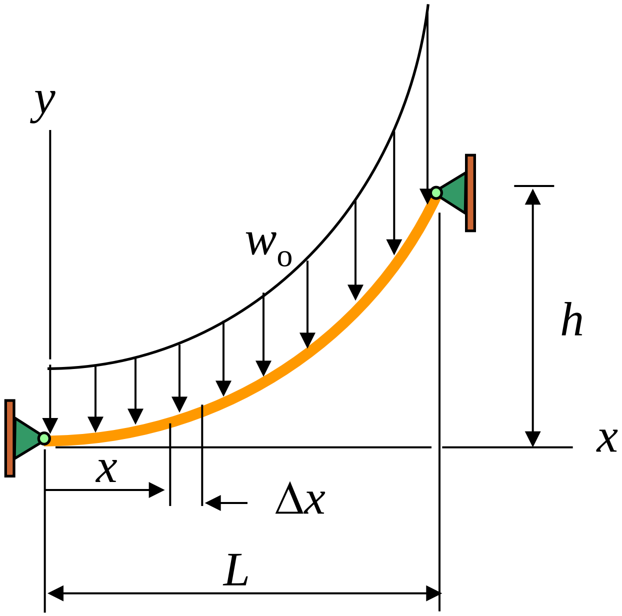


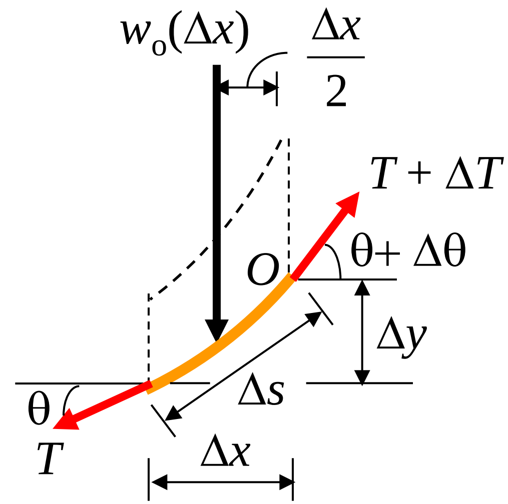
$$\boxed{T_o = \frac{w_o x^2}{2y}}$$

at $x = L$, $T = T_B = T_{\max}$

$$\boxed{T_{\max} = \sqrt{T_o^{\,2} + (w_o L)^2}}$$







$\xrightarrow{+}$ $\Sigma F_x = 0$: $\quad -T\cos\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$

$+\uparrow$ $\Sigma F_y = 0$: $\quad -T\sin\theta - w_o(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$

$\circlearrowleft +$ $\Sigma M_O = 0$: $\quad w_o(\Delta x)(\Delta x/2) - T\cos\theta\Delta y + T\sin\theta(\Delta x) = 0$

Dividing each of these equations by Δx and taking the limit as $\Delta x \rightarrow 0$, and hence $\Delta y \rightarrow 0$, $\Delta\theta \rightarrow 0$, and $\Delta T \rightarrow 0$, we obtain

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad \text{----------(1)}$$

$$\frac{d(T\sin\theta)}{dx} = w_o \qquad \text{----------(2)}$$

$$\frac{dy}{dx} = \tan\theta \qquad \text{----------(3)}$$

Integrating Eq. 1, where $T = F_H$ at $x = 0$, we have:

$$T\cos\theta = F_H \qquad \text{----------(4)}$$

Integrating Eq. 2, where $T \sin\theta = 0$ at $x = 0$, gives

$$T\sin\theta = w_o x \qquad \text{----------(5)}$$

Dividing Eq. 5 Eq. 4 eliminates T. Then using Eq. 3 , we can obtain the slope at any point,

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \qquad \text{----------(6)}$$

Performing a second integration with $y = 0$ at $x = 0$ yields

$$y = \frac{w_o}{2F_H} x^2 \qquad \text{----------(7)}$$

This is the equation of a *parabola*. The constant F_H may be obtained by using the boundary condition $y = h$ at $x = L$. Thus,

$$F_H = \frac{w_o L^2}{2h} \qquad \text{----------(8)}$$

Finally, substituting into Eq. 7 yeilds

$$y = \frac{h}{L^2} x^2 \qquad \text{----------(9)}$$

From Eq. 4 , the maximum tension in the cable occurs when θ is maximum; i.e., at $x = L$. Hence, from Eqs. 4 and 5,

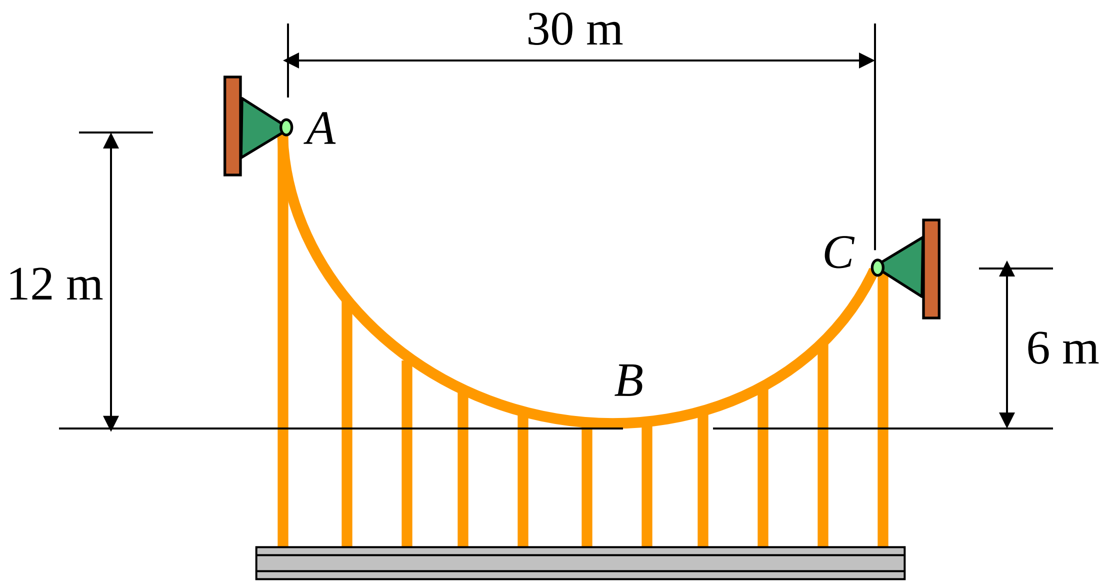
$$T_{\max} = \sqrt{{F_H}^2 + (w_o L)^2} \qquad \text{----------(10)}$$

Or, using Eq. 8 , we can express $T_{\max}$ in terms of w_o, i.e.,

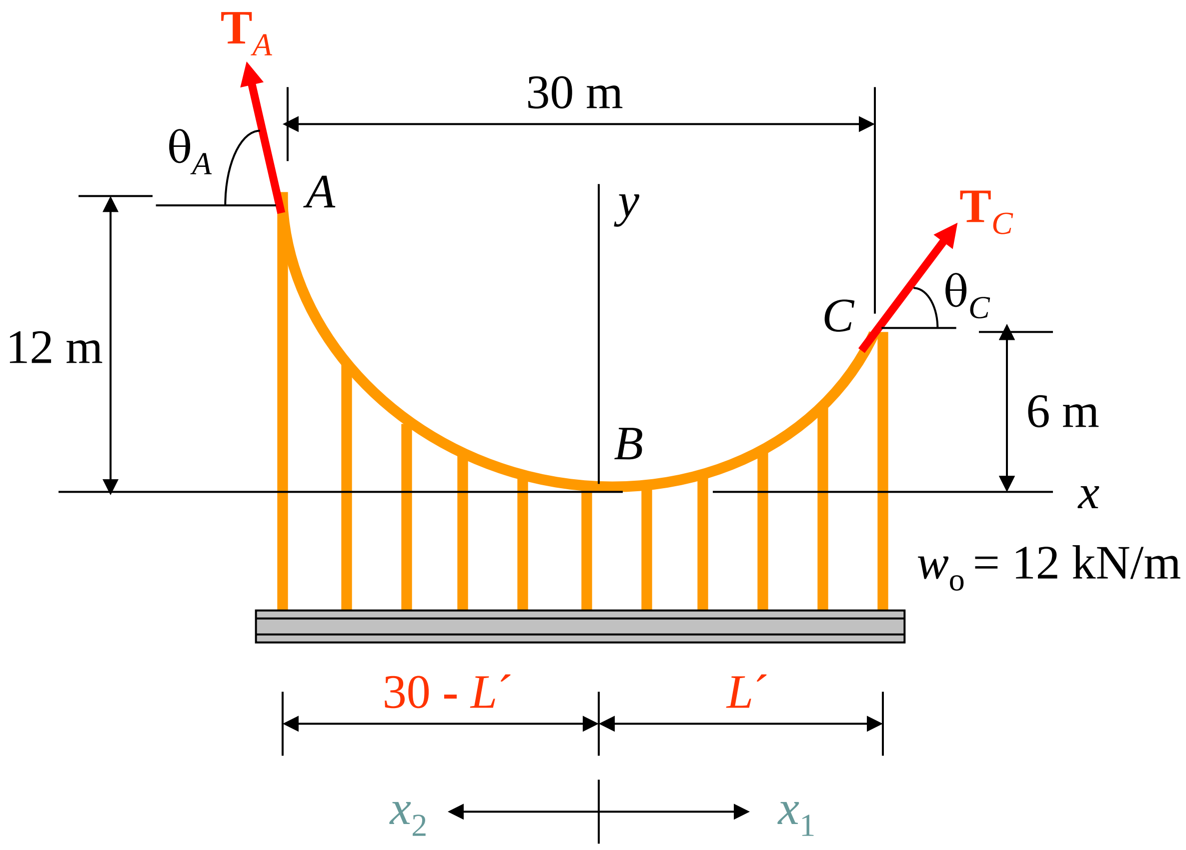
$$T_{\max} = w_o L \sqrt{1 + (L/2h)^2} \qquad \text{----------(11)}$$

Example 2

The cable shown supports a girder which weighs 12kN/m. Determine the tension in the cable at points *A*, *B*, and *C*.



SOLUTION



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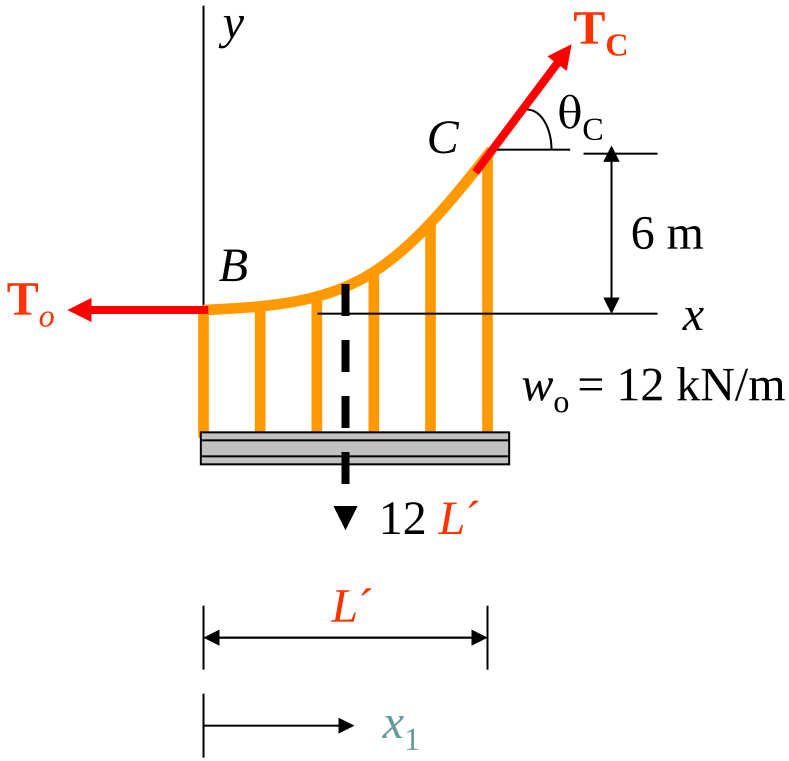

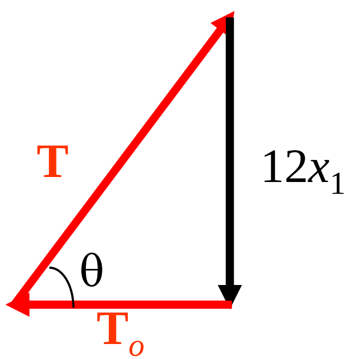

$$\frac{dy_1}{dx_1} = \tan\theta = \frac{12x_1}{T_o}$$

$$y_1 = \int \frac{12x_1}{T_o}\,dx_1$$

$$6 = \int_0^{L'} \frac{12x_1}{T_o}\,dx_1$$

$$6 = \left.\frac{12x_1^{\,2}}{2T_o}\right|_0^{L'} + \cancelto{0}{C_1}$$

$$6 = \frac{12L'^2}{2T_o}$$

$$\boxed{T_o = L'^2} \qquad \text{----------(1)}$$

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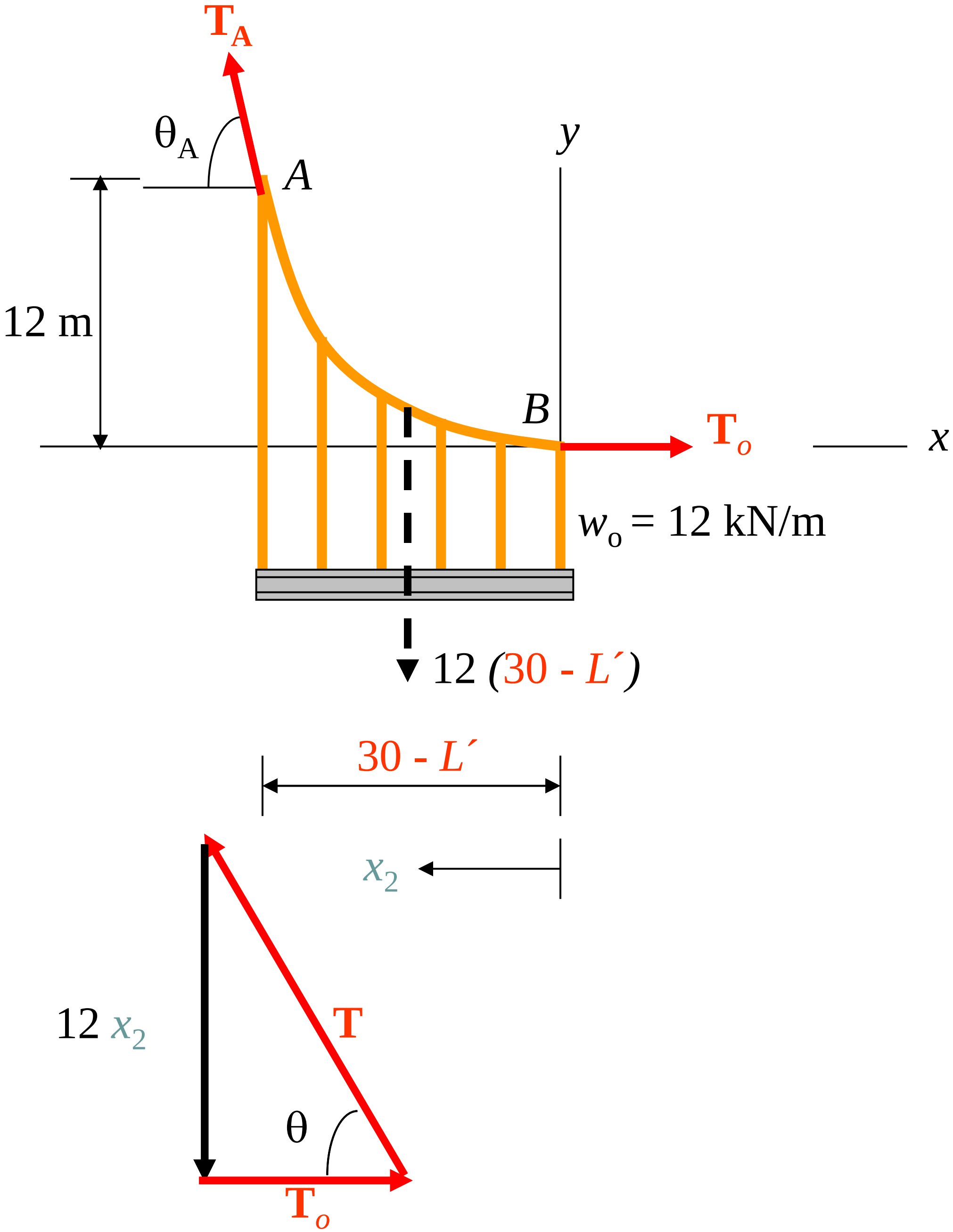


$$\frac{dy_2}{dx_2} = \tan\theta = \frac{12x_2}{T_o}$$

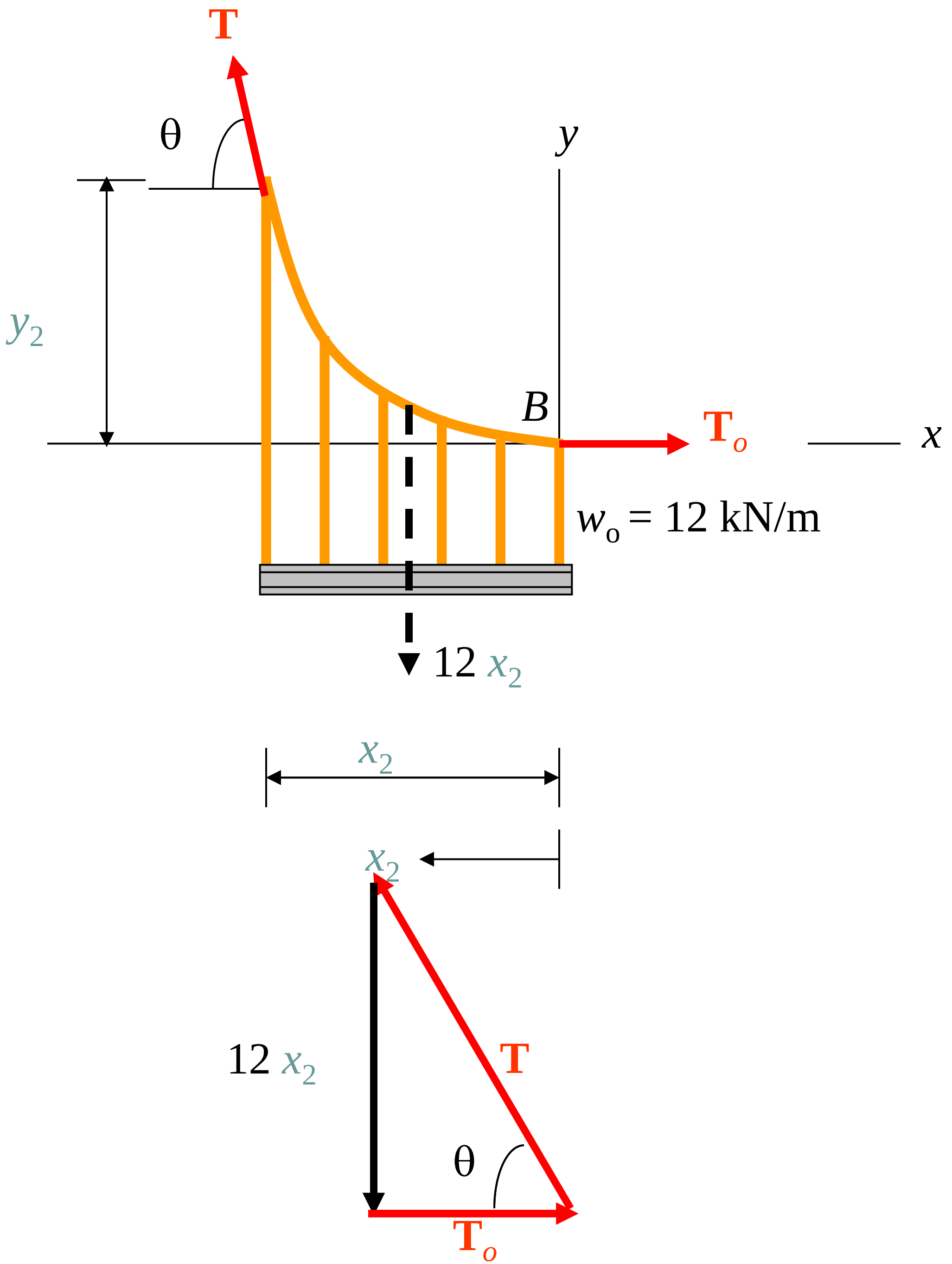
$$y_2 = \int \frac{12x_2}{T_o}\, dx_2$$

$$12 = \int_0^{(30-L')} \frac{12x_2}{T_o}\, dx_2$$

$$12 = \left.\frac{12x_2^{\,2}}{2T_o}\right|_0^{(30-L')} + \cancel{C_2}$$

$$\cancel{12} = \frac{\cancel{12}(30-L')^2}{2T_o}$$

$$\boxed{1 = \frac{(30-L')^2}{2T_o}} \quad \text{----------(2)}$$

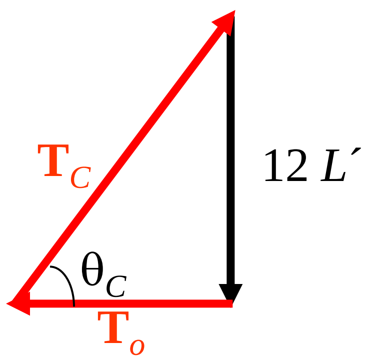

T
θ
y
y_2
B
T_o
x
w_o = 12 kN/m
12 x_2
x_2
x_2
12 x_2
T
θ
T_o

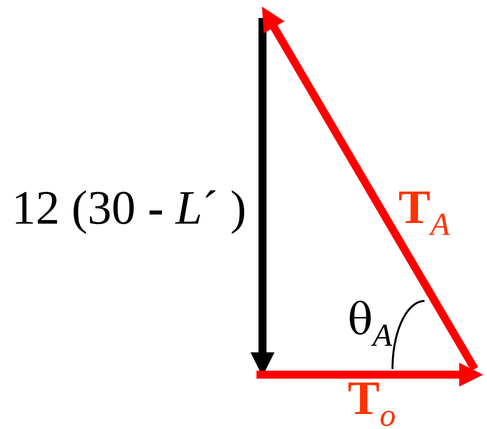
$$T_o = L'^2 \qquad \text{----------(1)}$$

$$1 = \frac{(30 - L')^2}{2T_o} \qquad \text{----------(2)}$$

From (1) and (2), $L' = 12.43$ m, $T_o = 154.50$ kN

$$T_B = T_o = 154.50 \text{ kN}$$





$$T_C = \sqrt{T_o^{\,2} + (12L')^2}$$

$$= \sqrt{(154.50)^2 + (12 \times 12.43)^2}$$

$$= 214.75 \text{ kN}$$

$$T_A = \sqrt{T_o^{\,2} + [12(30 - L')]^2}$$

$$= \sqrt{(154.50)^2 + [12(30 - 12.43)]^2}$$

$$= 261.39 \text{ kN}$$